STABILITY ANALYSIS OF FEEDBACK AMPLIFIERS BY MATRIX ALGEBRA METHODS

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by

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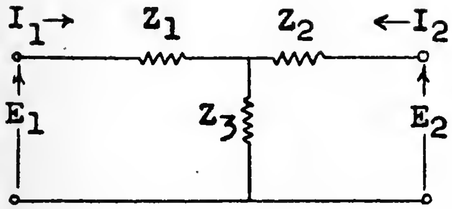
PREFACE

Matrix algebra has been used extensively in analysis of passive electrical circuits and grid-driven vacuum tube amplifiers, and thus has forged our intuition for the workings of passive reciprocal devices and active unilateral devices. The advent of the transistor, which is not entirely unilateral, has caused the need of wider knowledge of non-unilateral active circuits, which include feedback amplifiers. Examples of the above types of circuits are shown in Figure 1.

This paper presents methods of matrix algebra applied to the stability problem. The analysis must lead to the same result as any other method, and, although the amount of calculation may be the same as for other methods, the advantages of matrix algebra lie in arrangement and organization of the problem.

The writer wishes to thank Professor R. Kahal for suggesting the problem.

Passive



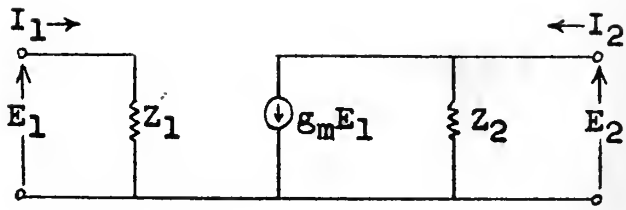
(Z_1, Z_2, Z_3 passive)

$Z_{11} = Z_1 + Z_3$

$Z_{12} = Z_{21} = Z_3$

$Z_{22} = Z_2 + Z_3$

Unilateral Active



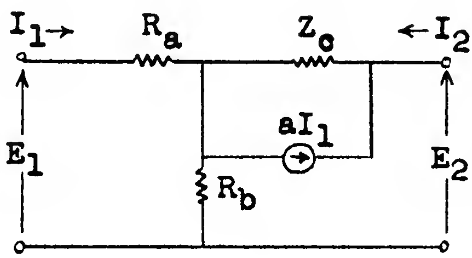
$Z_{11} = Z_1$

$Z_{12} = 0$

$Z_{21} = -g_m Z_1 Z_2$

$Z_{22} = Z_2$

Active



$Z_{11} = R_a + R_b$

$Z_{12} = R_b$

$Z_{21} = R_a + aZ_c$

$Z_{22} = R_b + Z_c$

Figure 1 - Active and Passive Circuits

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TABLE OF SYMBOLS AND ABBREVIATIONS

(Listed in the order of their use in the text)

Numbers in modified parenthesis, [ ], refer to the Bibliography. In general, matrices will be represented by parentheses, except where the omission of parentheses causes no ambiguity -- in such cases capital letters represent the matrix, i.e.,

$$Z = \begin{pmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{pmatrix}$$

det F is the determinant of the matrix F

$|Z|$ represents the absolute value of Z

n_t is the transpose of the matrix n, so that the elements a_{ij} of the matrix n equal the elements a_{ji} of n_t.

μ - the amplifier transmission characteristic

β - transmission characteristic through the feedback network

F - the return difference

T - the return ratio

CHAPTER I

INTRODUCTION

1. Early literature

The analysis of linear four-poles by the use of matrix algebra has received extensive treatment in the literature [2,5,11]. The treatment, at first restricted to passive networks obeying the reciprocity relation, was later extended to networks containing vacuum tubes [3,4,15] and gyrators [19]. Early literature regarding stability of networks failed to make use of matrix algebra [14]. Examples of the application of the transmission matrix to problems of stability are in papers by Jefferson [9], Sulzer [17], and Hinton [7], but these are limited merely to the derivation of the matrix network function corresponding to the Nyquist $\mu\beta$ criterion. Honnell [8] extended these methods and wrote a generalized criterion for stability of a closed tandem connection of two terminal-pair networks. More recently, Mason [13] indicated means of transforming amplifiers into stable forms and gave a relation between power gain and stability in feedback amplifiers.

CHAPTER II

STABILITY

1. General

An amplifier may be said to be stable if its response ultimately decays to zero after it has been subjected to a small impressed disturbance which itself dies out. If the response increases until limited by non-linearity, the amplifier is said to be unstable.

The determination of the stability of a non-unilateral amplifier leads directly to the analysis of the linear four-pole shown in Figure 2, which may include internal feedback loops.

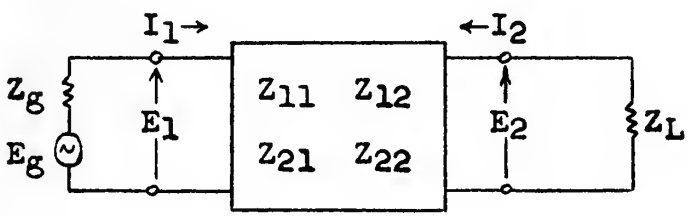


Figure 2 - The general non-unilateral amplifier

The characteristic equations may be written

$$\begin{aligned} E_1 &= Z_{11}I_1 + Z_{12}I_2 = E_g - Z_gI_1 \\ E_2 &= Z_{21}I_1 + Z_{22}I_2 = -Z_LI_2 \end{aligned} \qquad (1)$$

or, more concisely, in matrix form,

$$\begin{pmatrix} E_1 \\ E_2 \end{pmatrix} = \begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix} \cdot \begin{pmatrix} I_1 \\ I_2 \end{pmatrix} \qquad (2)$$

In general, we may say that Z_{12} and Z_{21} will not be equal, since the feedback amplifier includes active elements which are not bilateral, although there may be included passive, bilateral elements and active unilateral elements (i.e., vacuum tubes). Then, from equation (1),

$$I_1 = \frac{E_g(Z_{22} + Z_1)}{\Delta}$$

$$I_2 = \frac{-Z_{21}E_g}{\Delta} \qquad (3)$$

where $\Delta = Z_{11}Z_{22} - Z_{12}Z_{21} + Z_gZ_{22} + Z_LZ_{11} + Z_gZ_L$

We can see that, with zero impressed voltage, I_1 and I_2 can be other than zero when $\Delta = 0$, with Δ a function of $j\omega$. When Δ is considered a function of the more general frequency variable, $p = \sigma + j\omega$, so that the currents are of the form $e^{(\sigma + j\omega)t}$, it can be seen that for $\Delta = 0$, stability of the system requires $\sigma < 0$.

Thus, the location of the zeros of Δ are limited to the left half-plane, so that all the free modes of response are positively damped and so vanish with time. The presence of

a zero in the right half-plane is indicative of a negatively damped or runaway response.

CHAPTER III

VARIOUS STABILITY CRITERIA

1. Return-Difference and the Nyquist Diagram

In a single loop structure the fundamental quantity is the loop transmission, $\mu\beta$, which can be described as the voltage returned around the circuit when the loop is broken just behind any grid and a unit e.m.f. is applied to that grid. The factor $1 - \mu\beta$, which measures the reduction of the effect of parameter variations in a feedback amplifier, is closely related to the loop transmission.

Bode [1] used these quantities for a general definition of feedback, and defined the quantities Return Difference and Return Ratio. The two quantities are so closely related that the term feedback may be applied to both.

The return difference, F, is defined as the potential difference appearing between the grid node N and the grid G if this connection is broken and, while all other generators in the network are dead, a generator of unit e.m.f. is inserted between cathode K and the grid G. (see Figure 3).

The return ratio, T, is the voltage between the grid node N and the cathode K and is equal to (F-1). It will be remembered that this concept is applicable when opening the input to one grid. Thus, in an ordinary amplifier, $T = -\mu\beta$.

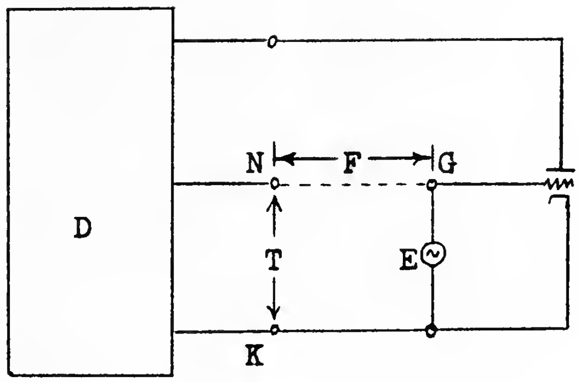


Figure 3 - Definition of Return Difference

The return difference, F, is shown to be equal to Δ / Δ^{o}, where Δ is the circuit determinant, and Δ^{o} is the circuit determinant with the grid input under question opened.

The requirements on Δ for stability of the structure were mentioned previously as being that the roots could not appear in the right half-plane. Evidently we wish to investigate the behavior of F in the right half-plane to determine the stability or instability of a circuit, but the actual operation of the circuit is of interest only at real frequencies, i.e., on the $j\omega$ -axis.

The Nyquist diagram is useful in an investigation of the behavior of the circuit Δ in the right half-plane in that it utilizes a theorem from the theory of functions of a complex variable, which states that:

> If a function f(z) is analytic, except for possible poles, within and on a given contour the

number of times the plot of f(z) encircles the origin of the f(z) plane in the positive direction, while z itself moves around the prescribed contour once in a clockwise direction, is equal to the number of poles of f(z) lying within the contour diminished by the number of zeros of f(z) within the contour, when each zero and pole is counted in accordance with its multiplicity. [1]

The concepts of return difference and return ratio are useful in determining the stability of a network in that the Nyquist diagram is essentially a plot of the values of F, a function of p, corresponding to values of p lying on the boundary of the right half of the p-plane.

Practical reasoning assumes F to be negligible for large values of p, since at high frequencies the circuit becomes capacitive (this is the case while p moves around the infinite semi-circle). The complete diagram then becomes a plot of only the real frequency values of F for $-j\omega < p < +j\omega$. (See Figure 4).

For the case of a single return loop, Δ^0 has no zeros in the right half-plane, and any zeros of Δ in the right half-plane will result in the plot of F encircling the origin; thus, the condition for stability is that the plot shall not encircle the origin.

Although the above description envisions a plot of F, in practise the diagram is plotted in terms of the return ratio, T, so that the critical point shifts one unit to the left to (-1,0) in the T diagram, or to (+1,0) in the $\mu\beta$ diagram.

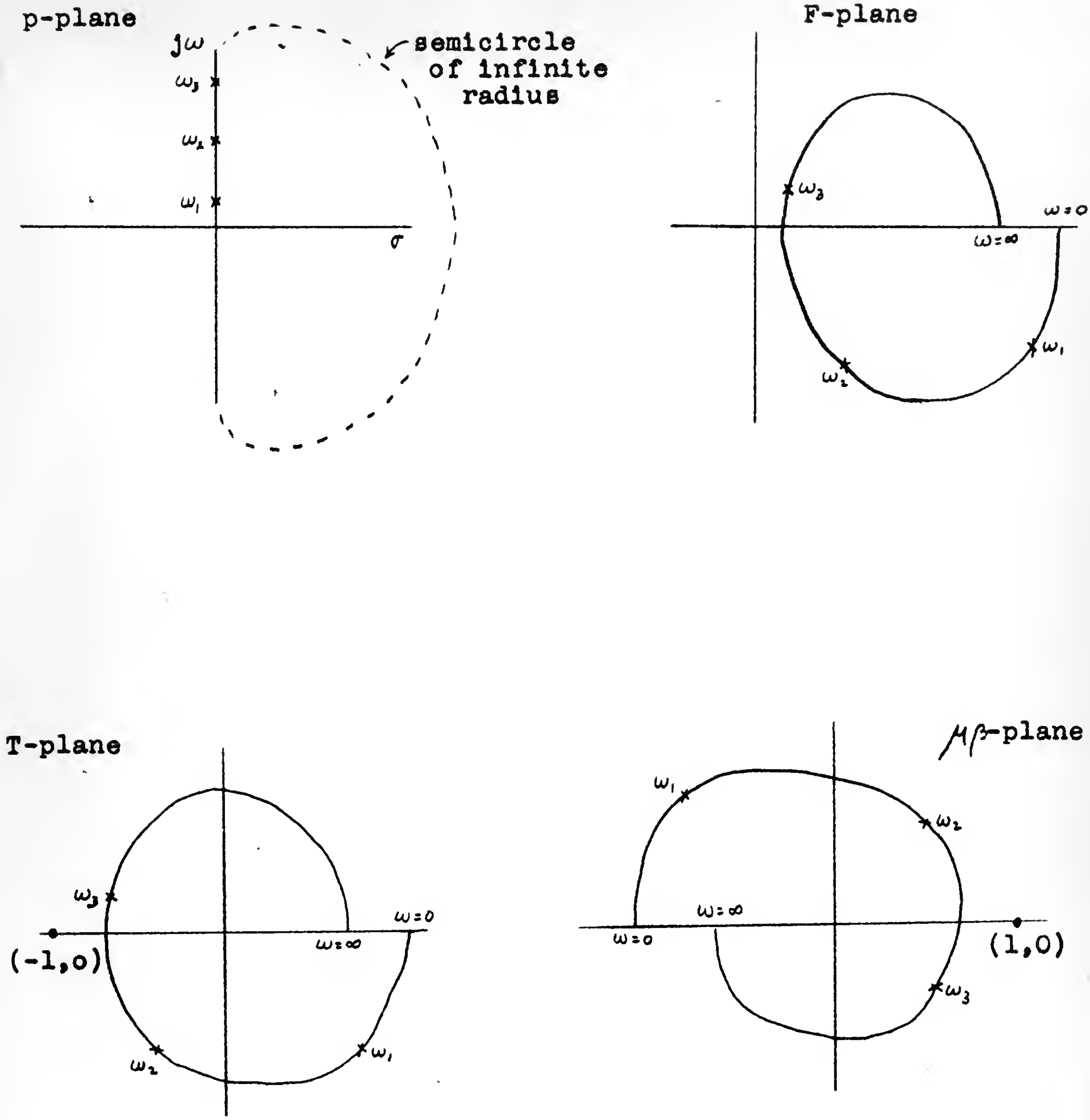


Figure 4 - The Nyquist diagram

2. The Return Difference Matrix

Bode's concept of the return difference was applicable to opening one grid input. This was extended to opening several grid inputs simultaneously by the use of matrices by Tasny-Tschiassny [18]. He defined the return difference matrix:

> The return difference matrix is defined by the statement that the voltage matrix
> $$V = FE$$
> appears between the grid nodes N and the grids G if the voltages, or currents, or both, generated by the input generators are made zero, and the considered elements are disconnected from the grid nodes N and connected to the cathodes K through zero impedance generators producing the voltage matrix E. [18]

The V's and E's are column matrices and F is a square matrix, so that from the definition above,

$$\begin{pmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{pmatrix} = \begin{pmatrix} F_{11} & F_{12} & \cdots & F_{1n} \\ F_{21} & \cdots & \cdots & \vdots \\ \vdots & & & \vdots \\ F_{n1} & \cdots & \cdots & F_{nn} \end{pmatrix} \cdot \begin{pmatrix} E_1 \\ E_2 \\ \vdots \\ E_n \end{pmatrix} \qquad (4)$$

The return ratio matrix, T, determines the corresponding voltages appearing between the grid nodes N and cathodes K, so that F and T are square matrices, the number of rows and columns being the number of considered elements.

Then, extending Bode's definition of T,

$$\begin{pmatrix} F_{11} & F_{12} & \cdots & F_{1n} \\ F_{21} & \cdots & \cdots & \vdots \\ \vdots & & & \vdots \\ F_{n1} & \cdots & \cdots & F_{nn} \end{pmatrix} = \begin{pmatrix} T_{11} & T_{12} & \cdots & T_{1n} \\ T_{21} & \cdots & \cdots & \vdots \\ \vdots & & & \vdots \\ T_{n1} & \cdots & \cdots & T_{nn} \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & \cdot \\ 0 & 0 & 1 & \cdots & \cdot \\ \cdot & & & & \cdot \\ 0 & \cdots & \cdots & \cdots & 1 \end{pmatrix} \tag{5}$$

or, more concisely,

$$F = T + I \tag{6}$$

where I is the appropriate unit matrix.

The elements of the return difference matrix are obtained by selecting, successively, voltage matrices E in which one element is equál to unity and all others are zero. The analysis must be carried out as many times as there are considered elements.

The condition for the network with considered grid inputs opened to be stable is shown to be

$$\det F \neq 0, \tag{7}$$

where det F is the determinant of the matrix F, so that F^{-1} exists.

Finally, the criterion for stability with considered elements closed is shown to be, for $\det F = 0$, $\sigma < 0$, where F is a function of the complex frequency $p = \sigma + j\omega$.

In terms of the Nyquist diagram, points on the $j\omega$ -axis in the p-plane are mapped into the det F plane, and the requirement for stability is that the origin not be encircled.

3. Honnell's generalized criterion.

Another approach to the stability criterion is shown by Honnell [8], who wrote a generalized criterion for stability of a closed tandem connection of two terminal-pair networks.

The transmission matrix of one two terminal-pair is

$$\begin{pmatrix} E_1 \\ I_1 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \cdot \begin{pmatrix} E_2 \\ -I_2 \end{pmatrix} \tag{8}$$

and the overall transmission matrix for n two terminal-pair networks connected in tandem as shown in Figure 5 is

$$\begin{pmatrix} E_1 \\ I_1 \end{pmatrix} = (a^1) \cdot (a^2) \cdot \,.\, (a^n) \cdot \begin{pmatrix} E_n \\ -I_n \end{pmatrix} \tag{9}$$

or, in more concise form

$$\begin{pmatrix} E_1 \\ I_1 \end{pmatrix} = (a^o) \cdot \begin{pmatrix} E_n \\ -I_n \end{pmatrix} \tag{10}$$

where (a^o) is the overall transmission matrix, and is obtained from the matrix product of the associated a^r matrices for the individual two terminal-pairs, taken in the proper

order.

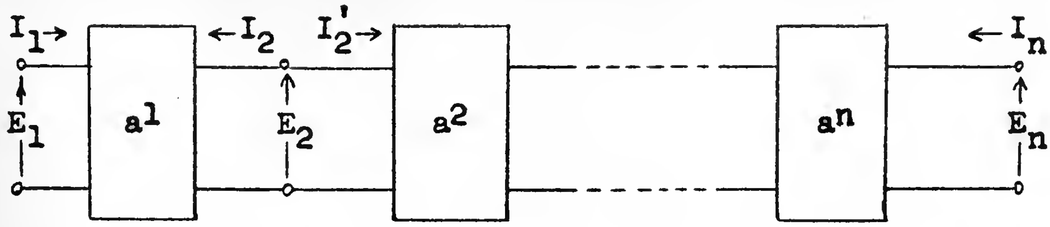


Figure 5 - Tandem connection of two terminal-pairs

The overall stability criterion, as a function of the complex frequency, p, is shown to be

$$A(p) = a^o_{11} + a^o_{22} - (\det a^o) - 1 = 0 \tag{11}$$

when the tandem connection is closed into a single loop. The roots of A(p), where p is the complex frequency $\sigma + j\omega$, must lie in the left half-plane, so that the requirement for stability is $\sigma < 0$.

The methods shown can establish that a circuit is stable or unstable, but give no indication of means of increasing the stability of a circuit on the verge of instability.

CHAPTER IV

THE UNILATERAL AMPLIFIER

1. General

It is obvious that an unstable amplifier can be stabilized by utilizing any expedient that will maintain the zeros of Δ in the left half-plane, or, in effect, alter the course of the Nyquist diagram and so avoid the enclosing of the critical point. This may be accomplished by altering the scale of the feedback, or by transforming the network into a form which will be absoltuely stable. With only a few simple restrictions, an amplifier which is unilateral (i.e., Z_{12} of the impedance matrix is equal to zero) is absolutely stable.

This chapter shows that a unilateral amplifier may be obtained with the aid of matrix algebra, and the conditions under which this amplifier is stable. The actual transformation is made with a coupling circuit which may be represented by matrices in various ways, and when the transformation has been made with a lossless coupling circuit, the new amplifier which appears at the terminals of the coupling circuit has the same power gain as the former. Once the amplifier is made unilateral, additional coupling may be considered for feedback and designed according to elementary feedback theory.[1]

Figure 6 illustrates how the coupling networks are connected to the amplifier. In many cases, the feedback network may be combined with the unilateralizing network.

[1]See Appendix II for application of matrices to feedback.

The amplifier is considered to have a common ground at each end, and is shown as a three terminal network rather than as in Figure 2. The characteristic equations are the same as for a two terminal-pair.

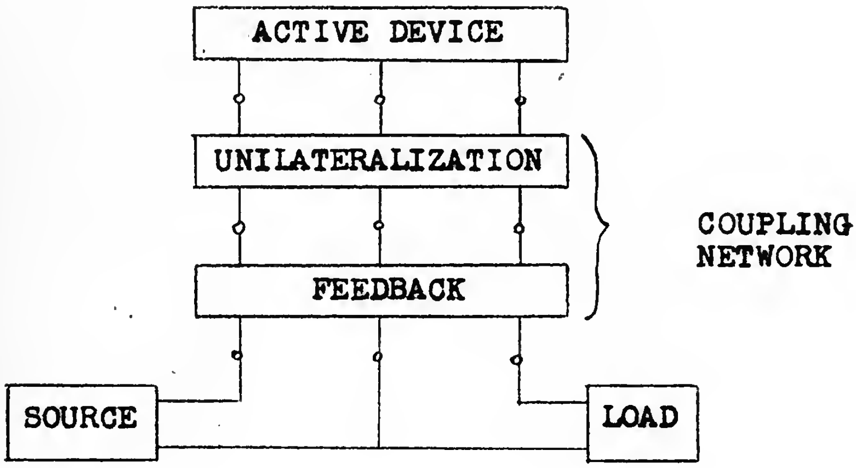


Figure 6 - Connection of coupling networks

2. Positive-Definiteness

If the matrix of the amplifier considered as a two terminal-pair is written

$$\begin{pmatrix} R_{11} + jX_{11} & R_{12} + jX_{12} \\ R_{21} + jX_{21} & R_{22} + jX_{22} \end{pmatrix}$$

it may be shown [12] that the system is stable for any passive termination at all frequencies if the following

conditions are satisfied:

$$R_{11} > 0$$

$$R_{22} > 0$$

$$4(R_{11}R_{22} + X_{12}X_{21})(R_{11}R_{22} - R_{12}R_{21}) - (R_{12}X_{21} - R_{21}X_{12})^2 > 0$$

These are the Gewertz conditions for stability in which the impedance matrix is described as being positive-definite.

If the amplifier is unilateral, i.e., $Z_{12} = 0$, these conditions reduce to

$$R_{11} > 0 \qquad (12)$$
$$R_{22} > 0$$

Thus, the conditions for stability of a unilateral amplifier reduce to rather simple forms, and it is for this reason that a means is sought to transform an amplifier which may be suspiciously near instability to a form which is absolutely stable.

The following section shows that a coupling network can always be found which will make the transformed matrix, Z', unilateral. The requirements for stability, then, are that R'_{11} and R'_{22} are both positive. When either R'_{11} or R'_{22} is negative, a new lossless coupling can be found which will make both R'_{11} and R'_{22} positive.

3. Lossless reciprocal transformations

If the amplifier of Figure 6 is non-unilateral, it is characterized by the equations written at terminals a:

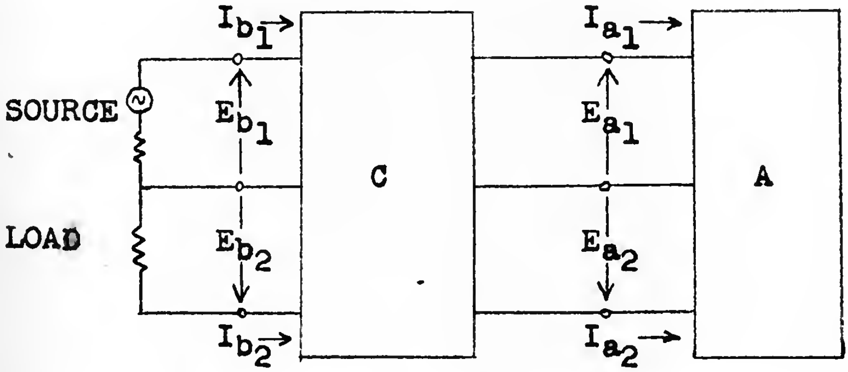


Figure 7 - The amplifier and coupling

$$\begin{pmatrix} E_{a_1} \\ E_{a_2} \end{pmatrix} = \begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix} \cdot \begin{pmatrix} I_{a_1} \\ I_{a_2} \end{pmatrix} \tag{13}$$

or, in more abbreviated form,

$$E_a = Z_{aa} I_a \tag{14}$$

In the same manner, the equations of the amplifier and coupling circuit may be measured at terminals b,

$$E_b = Z_{bb} I_b \tag{15}$$

In order to transform Z_{aa} into Z_{bb}, the equations of the coupling network C alone are written. The relations in the multi-terminal network C between E_1, E_2, I_1, and I_2

may be expressed by

$$\begin{pmatrix} E_1 \\ I_1 \end{pmatrix} = K \begin{pmatrix} E_2 \\ I_2 \end{pmatrix} \tag{16}$$

where the E's and I's are column matrices such that

$$E_1 = \begin{pmatrix} E_{b1} \\ E_{a1} \end{pmatrix} \qquad \text{etc.,}$$

and K is a 4 x 4 square matrix whose elements are independent of I and E. K, then, is given by the submatrices

$$K = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

and equation (16) is equivalent to

$$\begin{aligned} E_1 &= AE_2 + BI_2 \\ I_1 &= CE_2 + DI_2 \end{aligned} \tag{17}$$

The elements of A and D are dimensionless numbers, B is an impedance matrix, and C is an admittance matrix.

The characteristic equations of the coupling network can then be written in the form

$$\begin{pmatrix} E_a \\ E_b \end{pmatrix} = Z \begin{pmatrix} I_a \\ I_b \end{pmatrix} \tag{18}$$

where Z is an impedance matrix derived from K [16] such

that

$$\dot{Z} = \begin{pmatrix} AC^{-1} & AC^{-1}D - B \\ C^{-1} & C^{-1}D \end{pmatrix}$$

If the coupling circuit is reactive, equation (18) may take the form

$$\begin{pmatrix} E_a \\ E_b \end{pmatrix} = \begin{pmatrix} -jC_{aa} & jC_{ab} \\ -jC_{ba} & jC_{bb} \end{pmatrix} \cdot \begin{pmatrix} I_a \\ I_b \end{pmatrix} \tag{19}$$

where the C's are real; or, in expanded form,

$$\begin{aligned} E_a &= -jC_{aa}I_a + jC_{ab}I_b \\ E_b &= -jC_{ba}I_a + jC_{bb}I_b \end{aligned} \tag{20}$$

(Equation (19) has been expanded into equation (20), but equation (20) still represents abbreviated matrix notation from equations (13), (14), and (15)).

The transformation from Z_{aa} into Z_{bb} may be made by combining equations (14), (15), and (20), so that

$$\begin{aligned} (Z_{aa} + jC_{aa})I_a &= jC_{ab}I_b \\ (Z_{bb} - jC_{bb})I_b &= -jC_{ba}I_a \end{aligned} \tag{21}$$

from which,

$$(Z_{bb} - jC_{bb}) = C_{ba}(Z_{aa} + jC_{aa})^{-1} C_{ab} \tag{22}$$

The coupling circuit C is reciprocal since we labeled

it reactive, and matrix C is symmetric. Thus, C_{aa} and C_{bb} may be arbitrary, but symmetric, and $C_{ab} = (C_{ba})_t$, where the subscript t represents taking the transpose. Such a network, then, transforms the impedance matrix measured at the amplifier terminals into one at the coupling terminals by operations of the general form:

$$Z' - jx' = n(Z + jx)^{-1} n_t \tag{23}$$

where x and x' are arbitrary real symmetric matrices, n is an arbitrary real matrix, and n_t is the transpose of n.

It can be shown that lossless reciprocal coupling can accomplish any of the following operations[1]:

$$\begin{aligned} Z' &= Z + jx \qquad (\text{with } x_t = x) \\ Z' &= Z^{-1} \\ Z' &= nZn_t \end{aligned} \tag{24}$$

where x and n are real.

4. Unilateral Gain

Mason [18] shows that the parameter U, the unilateral gain, is given by:

$$U = \frac{\left| Z_{21} - Z_{12} \right|^2}{4(R_{11}R_{22} - R_{12}R_{21})} \tag{25}$$

and is invariant under the three operations of transform-

[1] See Appendix I.

ation given in (24) for lossless reciprocal coupling. For example, under the real transformation

$$Z' = nZn_t \tag{26}$$

consider the real part of (26):

$$R' = nRn_t \tag{27}$$

or

$$\begin{pmatrix} R'_{11} & R'_{12} \\ R'_{21} & R'_{22} \end{pmatrix} = \begin{pmatrix} n_{11} & n_{12} \\ n_{21} & n_{22} \end{pmatrix} \cdot \begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix} \cdot \begin{pmatrix} n_{11} & n_{12} \\ n_{21} & n_{22} \end{pmatrix} \tag{28}$$

whose determinant is

$$R'_{11}R'_{22} - R'_{12}R'_{21} = (n_{11}n_{22} - n_{12}n_{21})^2(R_{11}R_{22} - R_{12}R_{21}) \tag{29}$$

Also transpose equation (26) and subtract, yielding

$$Z' - Z'_t = n(Z - Z_t)n_t \tag{30}$$

whose determinant is

$$(Z'_{21} - Z'_{12})^2 = (n_{11}n_{22} - n_{12}n_{21})^2(Z_{21} - Z_{12})^2 \tag{31}$$

Substituting equations (29) and (31) into equation (25) shows that

$$U(Z') = U(nZn_t) = U(Z) \tag{32}$$

5. Example of a transformation to a unilateral form.

The non-unilateral amplifier of Figure 8(a) can be transformed into a unilateral form rather simply. The voltage represented by I_2R_{12} can be subtracted from the input terminals by means of a lossless transformer. Thus, V_1 in Figure 8(b) becomes, by proper choice of turns ratio,

$$V_1' = V_1 - \frac{R_{12}}{R_{22}}V_2 \tag{33}$$

so that the characteristic equations of the transformed amplifier are

$$\begin{aligned} V_1' &= R_{11}I_1 + R_{12}I_2 - \frac{R_{12}}{R_{22}}(R_{21}I_1 + R_{22}I_2) \\ &= \frac{R_{11}R_{22} - R_{12}R_{21}}{R_{22}}\, I_1 \\ V_2' &= (R_{21} - R_{12})I_1 + R_{22}I_2 \end{aligned} \tag{34}$$

or the transformed impedance matrix, Z', becomes

$$Z' = \begin{pmatrix} \dfrac{R_{11}R_{22} - R_{12}R_{21}}{R_{22}} & 0 \\ R_{21} - R_{12} & R_{22} \end{pmatrix} \tag{35}$$

The requirements for stability under this transformation are

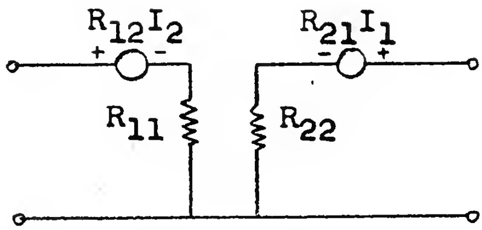
$$R_{11}' > 0$$
$$R_{22}' > 0$$

or

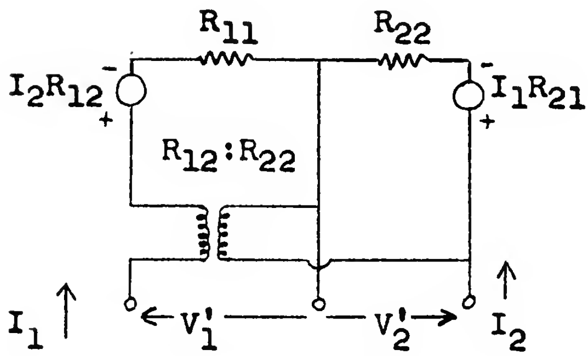
$$R_{11}R_{22} - R_{12}R_{21} > 0$$

$$R_{22} > 0 .$$

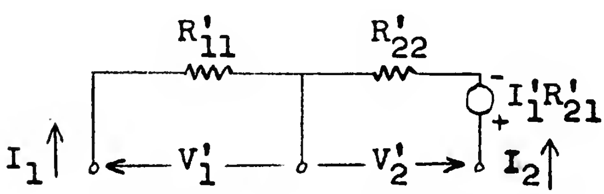
Figure 8



(a) A non-unilateral amplifier



(b) Transformed Amplifier



(c) Equivalent Unilateral Amplifier

6. Conclusions

Every consideration of the stability of a network has as its basis the fact that the roots of the circuit determinant must lie in the left half of the complex frequency plane. The investigation of stability, then, requires an investigation of the roots of Δ by one means or another, whether it be by calculations which may often be tiresome or impossible, or by means of the Nyquist diagram which signals the appearance of any roots in the right half-plane without their actual location being necessary.

The use of matrices in the first of these methods provides a powerful means of formalizing and organizing the investigation, even though the reduction of actual calculations may be minor. Honnell's criterion, for instance, signifies a great reduction in calculations once the overall transmission matrix is attained. Obtaining the overall transmission matrix, however, may require considerable calculation, so that the overall work may not be reduced with respect to any other method. The advantages of using matrix analysis then lies in the organizing of the problem.

The criteria discussed previously all have the disadvantage of the type of answer they give -- the circuit either is, or is not, stable. If a circuit lies on the verge of instability, there are generally a large number

of parameters which may be varied to secure a more stable network. The designer, then, usually has a choice in the number of factors he may vary, but at the same time he may also be faced with the problem of interaction between these parameters which might be the cause of some unwanted effect.

Essentially one might say, then, that the designer is faced with another design problem with the previous one acting only as a general basis from which to start.

The unilateral amplifier has been chosen in this paper as the basis for the new design problem because the conditions for absolute stability are not involved. Matrix algebra helps in transforming a non-unilateral amplifier into this unilateral form. Although its use does not actually synthesize the desired circuit, matrix algebra may act as a strong guide in determining what a transformation does, and lead our intuition to the network which will accomplish our aim.

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APPENDIX I

The impedance transformation does not always involve inversion of Z. Mason [13] demonstrates that by means of another transformation of Z' into Z" of the same general form as equation (17)

$$Z'' - jy'' = n'(Z' + jy')^{-1} n'_t$$

$$= n'(Z' - jx' + jx' + jy')^{-1} n'_t$$

$$= n' \left[n(Z + jx)^{-1} n_t + jx' + jy' \right]^{-1} n'_t$$

Choosing $x' + y' = 0$,

$$Z'' - jy'' = n' \left[n(Z + jx)^{-1} n_t \right]^{-1} n'_t$$

$$= n' n^{-1} (Z + jx) n_t^{-1} n'_t$$

which has the general form

$$Z'' - jy'' = n(Z + jx) n_t$$

APPENDIX II

FEEDBACK COUPLING

Once an amplifier is reduced to a normalized, unilateral, and hence positive-definite form, an additional lossless coupling, S, may be considered for feedback, and analyzed.

It is convenient to visualize the S terminals shown in Figure 9 as transmission lines of characteristic resistance unity. On this basis the incident waves S and reflected waves W' are related by the scattering matrix S [10]:

$$\begin{pmatrix} W_1' \\ W_2' \\ W_3' \\ W_4' \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{pmatrix} \cdot \begin{pmatrix} W_1 \\ W_2 \\ W_3 \\ W_4 \end{pmatrix} \tag{28}$$

Since we postulate matched loads to the lines, waves W_2 and W_4 must necessarily equal zero, and S becomes

$$S = \begin{pmatrix} S_{11} & 0 & S_{13} & 0 \\ S_{21} & 0 & S_{23} & 0 \\ S_{31} & 0 & S_{33} & 0 \\ S_{41} & 0 & S_{43} & 0 \end{pmatrix} \tag{29}$$

From equation (28) we can therefore extract the submatrix

$$\begin{pmatrix} W_2' \\ W_4' \end{pmatrix} = \begin{pmatrix} S_{21} & S_{23} \\ S_{41} & S_{43} \end{pmatrix} \cdot \begin{pmatrix} W_1 \\ W_3 \end{pmatrix} \tag{30}$$

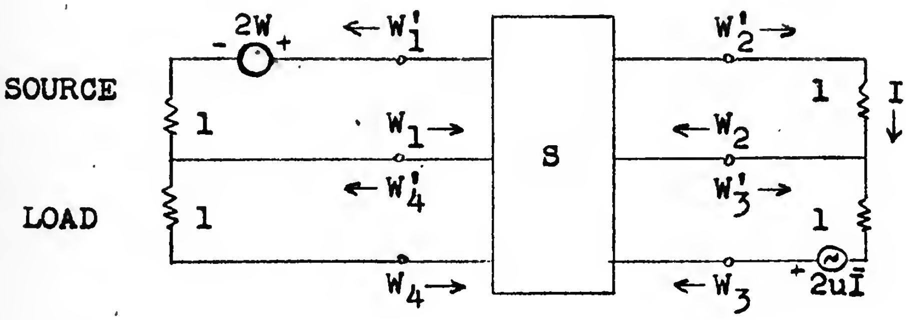

Figure 9 - Feedback coupling

or, more concisely,

$$W' = SW \tag{31}$$

For the greatest efficiency of transmission, we let the reflected waves W_1' and W_3' vanish. The coupling circuit is lossless and waves W and W' must carry the same power, or,

$$|W_2'|^2 + |W_4'|^2 = |W_1|^2 + |W_3|^2 \tag{32}$$

In matrix form, this is written

$$\begin{pmatrix} W_2' & W_4' \end{pmatrix} \cdot \begin{pmatrix} W_2'^* \\ W_4'^* \end{pmatrix} = \begin{pmatrix} W_1 & W_3 \end{pmatrix} \cdot \begin{pmatrix} W_1^* \\ W_3^* \end{pmatrix} \tag{33}$$

or, more concisely,

$$W'_t \; W'^* = W_t W^* \tag{34}$$

where * denotes the complex conjugate.

However, from equation (31)

$$W'_t \;\; W'^* = W_t S_t S^* W^* \tag{35}$$

where $W'_t = W_t S_t$, according to the reversal rule [11], so that the power conservation condition of equation (34) requires

$$S_t S^* = I, \text{ the appropriate unit matrix}$$

or

$$S_t = S^{*\,-1} \tag{36}$$

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algebra methods.